CYCLOTOMIC FIELDS PROBLEM SHEET 3

TOM LOVERING (LOVERING@FAS.HARVARD.EDU)

(1) The prime p is ramified in $\mathbb{Q}(\zeta_p)$. What is its ramification index e_p? Also compute this in $\mathbb{Q}(\zeta_{p^n})$. What do you conclude about f_p and r_p?
(2) Let $K = \mathbb{Q}(\zeta_q)$ with q an odd prime. For any $p \neq q$, we know $e_p = 1$. Show that $\sigma_p : \zeta \mapsto \zeta^p$ is a generator of the decomposition group, and use this to compute f_p and r_p.[1]
(3) Compute e_p, f_p, r_p for all primes p in $\mathbb{Q}(\zeta_n)$.
(4) Is 31 a square modulo 691? (if you think this should involve a long computation, think again!).
(5) Compute
$$1^9 + 2^9 + ... + 999999^9.$$
(6) For $a \in \mathbb{Z}$, and any $m \geq 0$, show that $a(a^m - 1)B_m$ and $a^m(a^m - 1)B_m/m$ are both integers.
(7) For $m \geq 3$ show that $|B_{2m+2}| > |B_{2m}|$.

[1] Hint: Use the Chinese remainder theorem.